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On determination of the large-$\frac{1}{x}$ gluon distribution at HERA

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Abstract

We discuss corrections to the Leading-LogQ^2 relationships between the gluon density $g(x,Q^2)$ and $F_L(x,Q^2)$, $\partial F_T(x,Q^2)/\partial \log Q^2$ in the HERA range of large $\frac{1}{x}$. We find that the above quantities probe the gluon density $g(x,Q^2_{T,L})$ at $Q^2_{T,L} = C_{T,L}Q^2$, with the Q^2-rescaling factors $C_T \approx 2.2$ and $C_L \approx 0.42$. The possibility of treating charm as an active flavour is critically re-examined.

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1 Introduction.

At large $\frac{1}{x}$, the gluon desnity $g(x,Q^2)$ is much higher than the density of charged partons $q(x,Q^2), \bar{q}(x,Q^2)$ (hereafter x is the Bjorken variable and Q^2 is the virtuality of the photon) and the photoabsorption will be dominated by interaction with the nucleon of the lightcone $q\bar{q}$ Fock states of the photon via the exchange by gluons (Fig. 1) or, alternatively, by the photon-gluon fusion $\gamma^* g \to q\bar{q}$. Consequently, the longitudinal structure function $F_L(x,Q^2)$ and the slope of the transverse structure function $F_T(x,Q^2) = 2xF_1(x,Q^2)$ become direct probes of the gluon structure function $G(x,Q^2) = xg(x,Q^2)$:

$$F_L(x,Q^2) = \frac{\alpha_S(Q^2)}{3\pi} \sum e_f^2 G(\xi_L x, Q^2)\,, \tag{1}$$

$$\frac{\partial F_T(x,Q^2)}{\partial \log(Q^2)} = \frac{\alpha_S(Q^2)}{3\pi} \sum e_f^2 G(\xi_T x, Q^2) \tag{2}$$

In Eqs. (1,2) $\alpha_S(Q^2)$ is the running strong coupling, e_f is the quark charge in units of the electron charge and the summation goes over the active flavours f. The suggestion of F_L as a probe (partonometer) of the gluon density is due to Dokshitzer [1], Eq. (1) was elaborated in [2]. Eq. (2) readily follows from formulas (6.37) and (6.34) of the Roberts' textbook [3] (see also [4]). Both equations were derived in the Leading-LogQ^2 approximation (LLQA), the x-rescaling factors $\xi_{T,L} \approx 2$ [2-4]. The emergence of the gluon-dominated scaling violations at large $\frac{1}{x}$ was clearly demonstrated in the recent QCD-evolution analysis of the NMC structure functions [5]. The obvious advantage of Eqs. (1,2) is that one does not need solving the coupled QCD-evolution equations for the gluon and (anti)quark densities.

As a byproduct of our analysis [6] of determination of the BFKL pomerons intercept from $F_L(x,Q^2), \partial F_T(x,Q^2)/\partial \log Q^2$ we have noticed the potentially important corrections to the LLQA. One obvious issue is which Q^2 is sufficiently large for charm to be treated as an active flavor, because for the (u,d,s) active flavours $\sum e_f^2 = \frac{2}{3}$ compared to $\sum e_f^2 = \frac{10}{9}$ if charm also were an active flavour. Our analysis [6] suggests that it is premature to speak of $N_f = 4$ active flavours unless $Q^2 \gtrsim (100-200)\mathrm{GeV}^2$. In [6] we also noticed that $F_L(x,Q^2)$ and $\partial F_T(x,Q^2)/\partial \log Q^2$ probe the gluon structure function $G(x,Q^2_{T,L})$ at different values of $Q^2_{T,L}$, which are both different from Q^2. Because of importance of determination of the gluon structure function, which is a fundamental quantity in the QCD parton model, in this paper we present an update of Eqs. (1,2)

which are valid also in the BFKL (Balitzkii-Fadin-Kuraev-Lipatov [7]) regime, i.e., beyond the LLQA.

At large $\frac{1}{x}$, deep inelastic scattering can be viewed as an interaction with the target nucleon of the lightcone $q\bar{q}$ Fock states of the photon via the exchange by gluons (Fig.1). The principal quantities are the probability densities $|\Psi_{T,L}(z,r)|^2$ for the $q\bar{q}$ Fock states with the transverse size $\vec{r}$ and the fraction z of photon's lightcome momentum carried by the (anti)quark, and $\sigma(x,r)$ - the total cross section of interaction of the $q\bar{q}$ colour dipole of transverse size r with the nucleon target [8,9]. This dipole cross section satisfies the generalized BFKL equation derived in [9,10] and is related to the differential density of gluons by the equation [9,11,12]

$$\sigma(x,r) = \frac{\pi}{3}\alpha_S(r)r^2 \int \frac{d^2\vec{k}}{k^2} \cdot \frac{4[1-\exp(i\vec{k}\,\vec{r})]}{k^2 r^2} \frac{\partial G(x_g,k^2)}{\partial \log k^2} \,. \tag{3}$$

The wave functions of the $q\bar{q}$ Fock states of the (T) transverse and (L) longitudinal photon were derived in [8] and read

$$|\Psi_T(z,r)|^2 = \sum e_f^2 |\Psi_T^{(f\bar{f})}(z,r)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_1^{N_f} e_f^2 \{[z^2+(1-z)^2]\varepsilon^2 K_1(\varepsilon r)^2 + m_f^2 K_0(\varepsilon r)^2\} \ , \tag{4}$$

$$|\Psi_L(z,r)|^2 = \sum e_f^2 |\Psi_T^{(f\bar{f})}(z,r)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_1^{N_f} 4e_f^2 \ Q^2 \, z^2(1-z)^2 K_0(\varepsilon r)^2 \ , \tag{5}$$

where $K_\nu(x)$ are the modified Bessel functions, $\varepsilon^2 = z(1-z)Q^2 + m_f^2$ and m_f is the quark mass. The resulting photoabsorption cross sections are equal to ([8], see also [13])

$$\sigma_T(\gamma^* N, x, Q^2) = \int_0^1 dz \int d^2\vec{r}\, |\Psi_T(z,r)|^2 \sigma(x,r) = \frac{2\alpha_{em}}{\pi} \sum_f e_f^2 \int_0^1 dz \int \frac{d^2\vec{k}}{k^4}$$
$$\int \frac{d^2\vec{\kappa}}{\vec{\kappa}^2+\varepsilon^2} \left\{ \frac{[z^2+(1-z)^2]\vec{k}^2 + m_f^2}{\vec{k}^2+\varepsilon^2} - \frac{[z^2+(1-z)^2]\vec{k}(\vec{k}+\vec{\kappa}) + m_f^2}{(\vec{k}+\vec{\kappa})^2+\varepsilon^2} \right\} \frac{\partial G(x_g,k^2)}{\partial \log k^2} \alpha_S(q^2) \,, \tag{6}$$

$$\sigma_L(\gamma^* N, x, Q^2) = \int_0^1 dz \int d^2\vec{r}\, |\Psi_L(z,r)|^2 \sigma(x,r) = \frac{2\alpha_{em}}{\pi} \sum_f e_f^2 \int_0^1 dz\, 4Q^2 z^2 (1-z)^2 \int \frac{d^2\vec{k}}{k^4}$$
$$\int \frac{d^2\vec{\kappa}}{\vec{\kappa}^2+\varepsilon^2} \left\{ \frac{1}{\vec{k}^2+\varepsilon^2} - \frac{1}{\vec{k}(\vec{k}+\vec{\kappa})+\varepsilon^2} \right\} \frac{\partial G(x_g,k^2)}{\partial \log k^2} \alpha_S(q^2) \,, \tag{7}$$

Here the running coupling $\alpha_S(q^2)$ enters the integrand at the largest relevant virtuality,

$$q^2 = \max\{\varepsilon^2 + \kappa^2, k^2\} \,, \tag{8}$$

and the density of gluons enters at

$$x_g = x(1 + \frac{M_t^2}{Q^2}) \tag{9}$$

where M_t is the transverse mass of the produced $q\bar{q}$ pair in the photon-gluon fusion $\gamma^* g \longrightarrow q\bar{q}$:

$$M_t^2 = \frac{m_f^2 + \vec{\kappa}^2}{1-z} + \frac{m_f^2 + (\vec{\kappa} + \vec{k})^2}{z}\,. \tag{10}$$

The flavour and Q^2 dependence of structure functions is concentrated in wave functions (4,5), whereas the dipole cross section $\sigma(x,r)$ (the differential gluon density $\partial G(x,k^2)/\partial \log k^2$ in the momentum representation) is universal for all flavours. The important virtue of the $(\vec{r},z)$ representation in (6,7) is the factorization of integrands as $|\Psi_{T,L}(z,r)|^2\sigma(x,r)$, which corresponds to the *exact* diagonalization of the diffraction scattering matrix in the $(\vec{r},z)$-representation. Furthermore, the above dipole-cross section representation (6,7) and wave functions (4,5) are valid also in the BFKL regime, i.e., beyond the LLQA, and allow an easy incorporation of the unitarity (absorption) corrections at large $\frac{1}{x}$ [9,11], with allowance for which Eq. (3) must rather be regarded as an operational definition of the gluon density beyond the LLQA. The x (energy) dependence of the dipole cross section $\sigma(x,r)$ comes from the higher $q\bar{q}g_1...g_n$ Fock states of the photon, i.e., from the QCD evolution effects, described at large $\frac{1}{x}$ by the generalized BFKL equation [9,10]. The structure functions are given by the familar equation $F_{T,L}(x,Q^2) = (Q^2/4\pi\alpha_{em})\sigma_{T,L}(x,Q^2)$. We advocate using F_L and $F_T = 2xF_1$, which have simpler theoretical interpretation compared to $F_2 = F_T + F_L$ which mixes interactions of the transverse and longitudinal photons. For the sake of completeness, we notice that, in the Born approximation the differential gluon density is related to the two-body formfactor of the nucleon $\langle N|\exp(i\vec{k}_1\vec{r}_1 + i\vec{k}_2\vec{r}_2)|N\rangle$ by the equation [12]

$$\frac{\partial G(x,k^2)}{\partial \log k^2} = \frac{4}{\pi}\alpha_S(k^2)(1 - \langle N|\exp(i\vec{k}(\vec{r}_1 - \vec{r}_2)|N\rangle\,, \tag{11}$$

and the limiting form of Eqs. (6,7) derived in [8] is obtained.

The leading contribution comes from values of $M_t^2 \sim Q^2$, so that $x_g \sim 2x$. Strictly speaking, the x-rescaling factors can not be determined within the Leading-Log$\frac{1}{x}$ approximation. From the practical point of view, when analysing F_L and $\partial F_T/\partial \log Q^2$, it is sufficient to use $x_g = \xi_{T,L}x$ with $\xi_{T,L} \approx 2$ as determined in [2-4]. In this communication we concentrate on corrections to the LLQA.

2 Active flavours and the onset of LLQA

The ratio $\sigma(x,r)/r^2$ is a smooth function of r. Similarly, $\partial G(x,k^2)/\partial \log k^2$ is a smooth function of k^2. Consequently, it is convenient to use the representations

$$F_T(x,Q^2) = \frac{1}{\pi^3}\int \frac{dr^2}{r^2}\frac{\sigma(x,r)}{r^2}\sum e_f^2 \Phi_T^{(f\bar{f})}(Q^2,r^2)\,, \tag{12}$$

$$\begin{aligned} F_L(x,Q^2) &= \frac{1}{\pi^3}\int \frac{dr^2}{r^2}\frac{\sigma(x,r)}{r^2}\sum e_f^2 W_L^{(f\bar{f})}(Q^2,r^2) \\ &= \frac{\alpha_S(Q^2)}{3\pi}\sum e_f^2 \int \frac{dk^2}{k^2}\Theta_L^{(f\bar{f})}(Q^2,k^2)\frac{dG(\xi_L x,k^2)}{d\log k^2}\,, \end{aligned} \tag{13}$$

$$\frac{\partial F_T(x,Q^2)}{\partial \log Q^2} = \frac{1}{\pi^3}\int \frac{dr^2}{r^2}\frac{\sigma(x,r)}{r^2}\sum e_f^2 W_T^{(f\bar{f})}(Q^2,r^2) \tag{14}$$

$$= \frac{\alpha_S(Q^2)}{3\pi}\sum e_f^2 \int \frac{dk^2}{k^2}\Theta_T^{(f\bar{f})}(Q^2,k^2)\frac{dG(\xi_T x,k^2)}{d\log k^2}\,, \tag{15}$$

where the weight functions $\Phi_T^{(f\bar{f})}$ and $W_{T,L}^{(f\bar{f})}$ are defined by

$$\Phi_T^{(f\bar{f})}(Q^2,r^2) = (\pi^2/4\alpha_{em})\int_0^1 dz\, Q^2 r^4 |\Psi_T^{(f\bar{f})}(z,r)|^2\,, \tag{16}$$

$$W_L^{(f\bar{f})}(Q^2,r^2) = (\pi^2/4\alpha_{em})\int_0^1 dz\, Q^2 r^4 |\Psi_L^{(f\bar{f})}(z,r)|^2\,, \tag{17}$$

$$W_T^{(f\bar{f})}(Q^2,r^2) = \frac{\partial \Phi_T^{(f\bar{f})}(Q^2,r^2)}{\partial \log Q^2}\,, \tag{18}$$

and the kernels $\Theta_{T,L}$ are given by

$$\Theta_{T,L}(Q^2,k^2) = \int \frac{dr^2}{r^2}\frac{\alpha_S(q^2)}{\alpha_S(Q^2)}\frac{4[1-J_0(kr)]}{(kr)^2}W_{T,L}(Q^2,r^2)\,. \tag{19}$$

Here $J_0(x)$ is the Bessel function and the running coupling $\alpha_S(q^2)$ enters at the largest relevant virtuality: $q^2 = \max\{k^2, C^2/r^2\}$, where $C \approx 1.5$ ensures the numerically similar results of calculations in the (r,z) and the momentum representations [8]. Since the BFKL equation is only known to the leading order, and the differences between the leading-order and next-to-leading order at large Q^2 are marginal [3], below we use the one-loop strong coupling $\alpha_S(k^2) = 4\pi/\beta_0 \log(k^2/\Lambda^2)$ with $\Lambda = 0.3$GeV. Here $\beta_0 = 11 - \frac{2}{3}N_f$, and in the numerical estimates we impose the infrared freezing $\alpha_S(k^2) \le \alpha_S^{(fr)} = 0.8$.

The conventional LLQA corresponds to

$$\Phi_T^{(f\bar{f})}(Q^2,r^2) = \theta(Q^2 - \frac{1}{r^2})\,, \tag{20}$$

$$\Theta^{(f\bar{f})}_{T,L}(Q^2,k^2) = \theta(Q^2-k^2) \tag{21}$$

and to the factoring out $\alpha_S(q^2) \approx \alpha_S(Q^2)$ from the integral (19). A derivation of the LLQA formulae (1,2) is based upon precisely these approximations. Our analysis of approximation (20) in [6] revealed a very slow onset of LLQA for the charmed quarks. Below we analyse in more detail an accuracy of Eq. (21) and of the LLQA relations (1,2).

Our results for $\Theta^{(f\bar{f})}_{T,L}(Q^2,k^2)$ are shown in Fig. 2. We find a very strong departure from the LLQA Eq. (21): i) the kernels $\Theta_{T,L}$ have a very broad diffuse edge, ii) the position of the diffuse edge is shifted compared to the naive expectation $k^2 = Q^2$, iii) the kernels $\Theta_{T,L}$ flatten at large Q^2, but the height of the plateau is different from unity, iv) the onset of LLQA for charm is very slow. Discussion of these effects is particulary simple in terms of the representation (19).

In order to facilitate the further discussion, we remind the salient features of weight functions $W^{(f\bar{f})}_{T,L}(Q^2,r^2)$ [6]. Firstly, at asymptotically large Q^2,

$$\int \frac{dr^2}{r^2} W^{(f\bar{f})}_{T,L}(Q^2,r^2) = 1\,. \tag{22}$$

Secondly, $W^{(f\bar{f})}_{T,L}(Q^2,r^2)$ are peaked at $r^2 = B_{T,L}/Q^2$, where

$$B_T \approx 2.3\,, \tag{23}$$

$$B_L \approx 11\,. \tag{24}$$

Therefore, $F_L(x,Q^2)$ and $\partial F_T(x,Q^2)/\partial \log Q^2$ probe $\sigma(x,r)$ at $r^2 = B_{L,T}/Q^2$. Notice a substantial departure from the naive LLQA expectation of $B_{T,L} \sim 1$. The width of these peaks is quite broad, $\Delta \log(Q^2r^2) \approx 3$ [6]. Furthermore, the function $f(x) = 4[1-J_0(x)]/x^2$, shown in Fig. 3, is similar to the step-function, but has a very broad diffuse edge. The position of the edge corresponds to an effective step-function approximation $f(x) = \theta(A_\sigma - x)$ with $A_\sigma \approx 10$, and leads to a dramatic deviation from the naive LLQA estimate $Q^2 \approx 1/r^2$ in the small-r limit of Eq. (3):

$$\sigma(x,r) \approx \frac{\pi^2}{3} r^2 \alpha_S(r) G(x, Q^2 \approx \frac{A_\sigma}{r^2})\,. \tag{25}$$

Now we discuss the results for $\Theta_{T,L}(Q^2,k^2)$ in the light of these observations.

The diffuse edge of kernels $\Theta_{T,L}$ originates from the two factors: a relatively large width of the weight functions $W_{T,L}(Q^2,r^2)$, and a broad diffuse edge of $f(kr) = 4[1-J_0(kr)]/(kr)^2$.

The kernels $\Theta_{T,L}$ flatten at $k^2 \ll Q^2$, but the height of the plateau $H_{T,L}^{(f\bar{f})}(Q^2)$ is significantly different from unity. At small Q^2 the height of the plateau is smaller than unity, which signals the sub-LLQA for the considered flavour. This effect is particularly important for heavy flavours (charm, bottom). For instance, for the charmed quarks $H_{T,L}^{(c\bar{c})}$ only very slowly rises with Q^2 reaching only $\sim .5$ at $Q^2 = 30\text{GeV}^2$, so that charm is an only marginally active flavor and for the charm conrtribution to the structure function the LLQA is badly broken unless $Q^2 \gtrsim (100-200)\text{GeV}^2$. For the b-quarks the height of the plateau is ≈ 0.3 at $Q^2 = 120\text{GeV}^2$ and ≈ 0.7 at $Q^2 = 480\text{GeV}^2$. The onset of the LLQA for heavy flavours is particularly slow in the case of the longitudinal structure function, because merely by gauge invariance the longitudinal cross section is suppressed, $\sigma_L/\sigma_T \propto Q^2/4m_f^2$ at $Q^2 \lesssim 4m_f^2$ (for instance, see [14]).

On the other hand, the excess over unity comes from $\alpha_S(q^2)/\alpha_S(Q^2) > 1$ in the integrand of Eq. (19). Of course, at the asymptotically large Q^2, the height of the plateau $H_{T,L}^{(f\bar{f})}(Q^2)$ tends to unity, because here the LLQA becomes accurate and one would have replaced $\alpha_S(q^2)$ in the integrand by $\alpha_S(\frac{1}{r^2}) \approx \alpha_S(Q^2)$. Closer inspection of Eq. (19) shows that, at the moderately large Q^2, besides the width of $W_{T,L}(Q^2, r^2)$ and the diffuse edge of $f(x)$, a large contributor to $\alpha_S(q^2) > \alpha_S(Q^2)$ is the large value of A_σ. In the region of Q^2 of the practical interest for the HERA experiments, the excess of $H_{T,L}^{(f\bar{f})}(Q^2)$ over unity is particularly large for the light quarks.

At sufficiently high $Q^2 \gtrsim 4m_f^2$, the position of the diffuse edge of $\Theta_{T,L}(Q^2, k^2)$ in the natural variable k^2/Q^2 is approximately flavour-independent. Notice, that the diffuse edge is definitely shifted towards positive $\log(k^2/Q^2) = \log(C_T) \sim 1$ for the slope of the transverse structure function, and for the longitudinal structure function the position of the edge is shifted towards negative $\log(k^2/Q^2) = \log(C_L) \sim -1$. We find strong departure from the LLQA assumption $C_{T,L} = 1$.

3 Measuring the gluon distribution

The differential gluon structure function $\partial G(x, Q^2)/\partial \log Q^2$ is a slow and smooth function of $\log Q^2$. For this reason, we can quantify the shift of the diffuse edge and the height of the plateau of global kernels $\Theta_{T,L}(Q^2, k^2)$ for $N_f = 5$ flavours (u, d, s, c, b) in terms of the effective

step-function parameterization

$$\Theta_{T,L}(Q^2,k^2) = \frac{9}{11}\sum_{f=1}^{5} e_f^2 \Theta_{T,L}^{(f\bar{f})}(Q^2,k^2) = \frac{9}{11} H_{T,L}(Q^2)\theta(C_{T,L}(Q^2)Q^2 - k^2)\,. \tag{26}$$

The so-determined height $H_{T,L}(Q^2)$ of the effective step-function and the Q^2-rescaling factor $C_{T,L}(Q^2)$ are shown in Fig. 4. They enter the modified relationships (1,2) as follows:

$$F_L(x,Q^2) = \frac{\alpha_S(Q^2)}{3\pi}\frac{11}{9} H_L(Q^2) G(\xi_L x, C_L(Q^2)Q^2) \tag{27}$$

$$\frac{\partial F_T(x,Q^2)}{\partial \log(Q^2)} = \frac{\alpha_S(Q^2)}{3\pi}\frac{11}{9} H_T(Q^2) G(\xi_T x, C_T(Q^2)Q^2) \tag{28}$$

The viable approximations for the kinematical range of the HERA experiments are $C_T(Q^2) = 2.2$ and $C_L(Q^2) = 0.42$. Because of variations of the factor $\alpha_S(q^2)/\alpha_S(Q^2)$ in the integrand of (19), these results for $C_{T,L}$ slightly differ from, but are close to, the estimate $C_{T,L}(Q^2) \sim B_{T,L}/A_\sigma$.

In the case of the slope of the transverse structure function, there is a significant numerical cancellation of effects of the suppressed, sub-LLQA contribution of heavy flavours, and of the enhanced height of the plateau for light flavours. Because of this numerical conspiracy, despite the charm not being an active flavour at all, we find $H_T(Q^2 > 10\mathrm{GeV}^2) = 1$ to better than 10 per cent accuracy in the kinematical range of HERA. For this reason, the recent estimate [15] of the gluon structure function from the H1 data on the scaling violation, using the the $N_f = 4$ LLQA formula (2), must be regarded as numerically reliable. For the longitudinal structure function this cancellation is less complete. The residual departure of $H_L(Q^2)$ from unity is a monotonic function of Q^2 and remains quite substantial in the kinematical range of HERA .

The accuracy of relations (27,28) can be checked computing first the structure functions for certain parametrization of the gluon density, and then comparing the input gluon density with the ouptut from (27,28). We have performed such a test for the gluon densities [11,12] and [16], both of which give a good description of the HERA data on the proton structure function [17]. For the derivative of the transverse structure function, the input/output comparison suggests that the accuracy of the above procedure is as good as 5 per cent at $Q^2 > 10\mathrm{GeV}^2$ in the whole HERA range of x. For the longitudinal structure function, the input/output agreement is better than 10 per cent at $x > 10^{-4}$ and $Q^2 > 10\mathrm{GeV}^2$, but gets worse at smaller x and not so large Q^2. At $x = 10^5$ the 10 per cent agreement only holds at $Q^2 > 30\mathrm{GeV}^2$. These estimates of the

accuracy of relations (27, 28) can easily be improved with the advent of high accuracy data on the gluon structure functions from the HERA experiments. Small corrections from the $z - r^2$ correlations in the wavefunctions (4.5) and/or the $x_g - (\vec{k}, \vec{\kappa})$ correlations implied by Eqs. (9,10), can also be easily included should the accuracy of the data require that.

4 Conclusions

We re-examined determination of the gluon structure function $G(x, Q^2)$ from the longitudinal structure function $F_L(x, Q^2)$ and from the slope $\partial F_T(x, Q^2)/\partial \log Q^2$ of the transverse structure function, and derived new relationships Eq. (27) and Eq. (28). In the range of Q^2 of the interest for HERA experiments, charm is only marginally active flavour. None the less, because of the numerical conspiracy of corrections to LLQA for the light and heavy flavour contributions, the overall normalization factor $H_T(Q^2) \approx 1$ as if all the $N_f = 5$ flavours (u, d, s, c, b) were active. Here the major effect is the Q^2-rescaling $C_T \approx 2.2$. We find that the onset of LLQA for the longitudinal structure function is much slower than for the transverse structure function. Numerical experiments suggest that our relationships (27,28) have $\lesssim 10\%$ accuracy.

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Figure captions

Fig.1 - Leading QCD subprocesses at large $\frac{1}{x}$.

Fig.2 - The kernels $\Theta_{T,L}(Q^2, k^2)$ for the light quarks (u, d), the charmed quark and the global kernel for $N_f = 5$ flavours (u, d, s, c, b). The dot-dashed, dashed, solid, double-dot-dashed and dotted curves are for $Q^2 = 0.75$, 2.5, 8.5, 30, 480GeV^2, respectively.

Fig.3 - The function $f(x) = 4[1 - J_0(x)]/x^2$.

Fig.4 - The height of the plateau $H_{T,L}(Q^2)$ and the Q^2-rescaling factor $C_{T,L}(Q^2)$ as a function of Q^2.

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